NOTE ON CAUCHY'S INTEGRAL TEST.

By M. B. PORTER, University of Texas.

It does not seem to have been generally noticed that series which consist of a monotonic decreasing sequence of terms of the same sign lend themselves to calculation as readily as those whose signs alternate provided the integral $\int_n^\infty u_n dn$ is sufficiently simple. We have at once, if the terms are all positive,

$$\int_n^\infty u_n dn \leq u_n + u_{n+1} + u_{n+2} + \ldots \text{etc.} \leq u_n + \int_n^\infty u_n dn.$$

Thus if to the sum of the first $n-1$ terms we add $\frac{u_n}{2} + \int_n^\infty u_n dn$, our approximation will not differ from the sum of the series by more than $u_n/2$, that is, by less than half of the first term we omit.

Thus to calculate the series $\sum_1^\infty n^{-2}$ the correction after $n-1$ terms is $\frac{1}{2n^2} + \frac{1}{n}$, and the calculated value is in error not more than $\frac{1}{2n^2}$. This fact can be used to calculate with a minimum of labor such slowly convergent series as those represented by the logarithmic scales of DeMorgan.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

346. Proposed by E. B. ESCOTT, Professor of Mathematics, University of Michigan.

Solve completely by quadratics:

$$\frac{2x}{1-x^2} = y; \quad \frac{2y}{1-y^2} = z; \quad \frac{2z}{1-z^2} = w; \quad \frac{2w}{1-w^2} = x.$$

Solution by the PROPOSER.

Let $x = \tan\phi$, then $y = \tan 2\phi$, $z = \tan 4\phi$, $w = \tan 8\phi$, $x = \tan 16\phi = \tan\phi$, $\tan 16\phi - \tan\phi = 0$, or $\frac{\sin 15\phi}{\cos 16\phi . \cos\phi} = 0$.